3/6/18 "Too many of us are not living our dreams because we are living our fears."-Les Brown

HW: "Systems of Linear Equations" Homework #1-4
Test 2 on Wednesday 3/14

AIM: How do we solve a system of equations?

Warm Up:

Solve the following systems of equations algebraically by either substitution or elimination:

1. $+$ $\begin{aligned} 2x - \cancel{y} &= -1 \\ 2x + \cancel{y} &= -7 \end{aligned}$

$$\frac{4x}{4} = \frac{-8}{4}$$

$$x = -2$$

$$\boxed{\begin{aligned} x &= -2 \\ y &= -3 \end{aligned} \quad \text{OR} \quad (-2, -3)}$$

To find: plug x in to either

$$2(-2) - y = -1$$

$$-4 - y = -1$$

$$+4 \qquad +4$$

$$-y = 3$$

$$y = -3$$

$$2x - y = -1$$
$$2x + y = -7 \longrightarrow y = -7 - 2x$$

$$2x - (-7 - 2x) = -1$$
$$2x + 7 + 2x = -1$$
$$4x + 7 = -1$$
$$\frac{4x}{4} = \frac{-8}{4} \quad x = -2$$

2. $2x+2y=3$
$x=4y-1$

Substitution

$2(4y-1)+2y=3$
$8y-2+2y=3$
$10y-2=3$
$+2 \quad +2$
$\frac{10y}{10}=\frac{5}{10}$
$y=\frac{1}{2}$

Find x:
$x=4y-1$
$x=4(\frac{1}{2})-1$
$x=2-1$
$x=1$

$x=1$
$y=\frac{1}{2}$

3. $2(x-2y=3)$
$-2x+4y=1$

$2x-4y=6$
$+ \quad -2x+4y=1$
$0+0=7$
$0=7$

Not true

$\boxed{\text{NO SOLUTION}}$

4. $-2(2x-y=1) \rightarrow -4x+2y=-2$
$4x-2y=2 \rightarrow + \quad 4x-2y=2$
$0+0=0$

True $\rightarrow$ $0=0$

$\boxed{\text{Infinite Solutions}}$

5. $3x+2y=2$
$5x+7y=-4$

Possibilities

1) One unique solution (consistent)
2) No Solution (Parallel Lines) (Inconsistent)
3) Infinite Solutions (Equations are the same line) (Dependent)

6. $3x+2y=-9$
$2x+y=-7$

You should be very familiar with solving two-by-two systems of linear equations (two equations and two unknowns). In this lesson, we will extend the method of **elimination** to linear systems of three equations and three unknowns. These linear systems serve as the basis for a field of math known as **Linear Algebra**.

7. Consider the three-by-three system of linear equations shown below. Each equation is numbered in this first exercise to help keep track of our manipulations.

$$\begin{array}{ll} (1) & 2x+y+z=15 \\ (2) & 6x-3y-z=35 \\ (3) & -4x+4y-z=-14 \end{array}$$

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?

1 $2x+y+z=15$
+
2 $6x-3y-z=35$
4 $8x-2y=50$

1 $2x+y+z=15$
+
3 $-4x+4y-z=-14$
5 $-2x+5y=1$

(b) Use this new two-by-two system to solve the three-by-three.

4 $8x-2y=50$
5 $4(-2x+5y=1) \rightarrow -8x+20y=4$

$\cancel{8x}-2y=50$
+ $-\cancel{8x}+20y=4$

$\frac{18y}{18}=\frac{54}{18}$

$y=3$

To find x plug the y into Eq 4 or 5

$-2x+5(3)=1$
$-2x+15=1$
$-2x=-14$
$x=7$

To find z we plug x and y into Eq 1 or 2 or 3

$2x+y+z=15$
$2(7)+3+z=15$
$14+3+z=15$
$17+z=15$
$z=-2$

$\boxed{\begin{array}{l} x=7 \\ y=3 \\ z=-2 \end{array}}$

$\boxed{(7,3,-2)}$

Just as with two by two systems, sometimes three-by-three systems need to be manipulated by the **multiplication property of equality** before we can eliminate any variables.

8. Consider the system of equations shown below. Answer the following questions based on the system.

$$\begin{aligned} ①\quad 4x + y - 3z &= -6 \\ ②\quad -2x + 4y + 2z &= 38 \\ ③\quad 5x - y - 7z &= -19 \end{aligned}$$

(a) Which variable will be easiest to eliminate? Why? Use the multiplicative property of equality and elimination to reduce this system to a two-by-two system.

① + ③

$$\begin{aligned} 4x + y - 3z &= -6 \\ 5x - y - 7z &= -19 \\ \hline ④\quad 9x - 10z &= -25 \end{aligned}$$

① $4(4x + y - 3z = -6) \Rightarrow 16x + \cancel{4y} - 12z = -24$

② $-2x + 4y + 2z = 38$ ⊖ $-2x + \cancel{4y} + 2z = 38$

⑤ $18x - 14z = -62$

(b) Solve the two-by-two system from (a) and find the final solution to the three-by-three system.

④ $-2(9x - 10z = -25) \longrightarrow -\cancel{18x} + 20z = 50$

⑤ $18x - 14z = -62 \longrightarrow$ (+) $\cancel{18x} - 14z = -62$

$$\begin{aligned} 6z &= -12 \\ z &= -2 \end{aligned}$$

To find x:

$$\begin{aligned} 18x - 14(-2) &= -62 \\ 18x + 28 &= -62 \\ -28 \quad & \quad -28 \\ \hline \frac{18x}{18} &= \frac{-90}{18} \\ x &= -5 \end{aligned}$$

To Find y: using ①

$$\begin{aligned} 4x + y - 3z &= -6 \\ 4(-5) + y - 3(-2) &= -6 \\ -20 + y + 6 &= -6 \\ -14 + y &= -6 \\ +14 \quad & \quad +14 \\ \hline y &= 8 \end{aligned}$$

$$\boxed{\begin{aligned} x &= -5 \\ y &= 8 \\ z &= -2 \end{aligned}}$$

9. Solve the system of equations shown below. Show each step in your solution process.

$$\begin{aligned} 4x - 2y + 3z &= 23 \\ x + 5y - 3z &= -37 \\ -2x + y + 4z &= 27 \end{aligned}$$